Towards Evaluation of Stringy Non-Perturbative Effects

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Abstract

We report on progress towards evaluation of stringy non-perturbative effects, using a two dimensional effective field theory for matrix models. We briefly discuss the relevance of such effects to models of dynamical supersymmetry breaking.

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1 Introduction

Supersymmetry breaking, particularly in the framework of string theory, is an interesting unsolved theoretical problem whose solution may have observable low-energy consequences accessible to future laboratory experiments. It is widely believed that dynamical supersymmetry breaking in string theory occurs through non-perturbatively induced interactions. At present it is possible to evaluate and control non-perturbative interactions in the low-energy field theory approximation to string theory which are typically of strength $exp(-1/g_{string}^2)$, where g_{string} is the string coupling constant. These interactions are an essential ingredient in models of supersymmetry breaking in string theory, as described, for example, in Taylor's talk.

However, it has been known for a while [1], that in addition to non-perturbative interactions of strength $exp(-1/g_{string}^2)$ there are also the so called "stringy non-perturbative effects" of strength $exp(-1/g_{string})$. Recently, some proposals for the source of stringy non-perturbative effects were put forward, one proposal [2] is that the their source are certain "D-instantons", associated with disconnected world-sheet holes and another proposed source [3] are type II string solitons of mass $1/g_{string}$.

Our proposal [4] is that stringy non-perturbative effects are associated with classical solutions for which the string coupling parameter varies in space-time and becomes strong in some region. In the space-time region of strong coupling, new degrees of freedom are important and are the source of stringy non-perturbative effects. Our method of calculation of these effects is based on a two dimensional effective field theory approach, and therefore can be applied in cases in which the effective dynamics in the strong coupling region is two-dimensional.

At the moment, the relationship between the different proposed sources is unclear, but the possibility that some or all have a common origin is very interesting. We will have nothing to add on this subject in this talk, rather we describe in some detail our approach.

2 Effective Two Dimensional Theory

Collective field theory [5] for bosonic $d = 1$ matrix models [6] is written in terms of the density of matrix eigenvalues, $\partial_x \varphi = \sum_{i=1}^N \delta(x - \lambda_i(t))$, where $\lambda_i(t)$ are the matrix eigenvalues, and the dimension of the matrix N, is very large. The (Euclidean) action of collective field theory is written in terms of the collective field φ, and is given by

$$S_E[\varphi] = \int dx dt \left\{ \frac{\dot{\varphi}^2}{2\varphi'} + \frac{\pi^2}{6}\varphi'^3 + \frac{1}{2}(\frac{1}{\omega g} - \omega^2 x^2)\varphi' \right\}. \tag{1}$$

The static, high density solution of the equation of motion derived from the action (1) is denoted by $\phi_0 \equiv \partial_x \varphi_0$, and is given by the simple expression $\phi_0 = \omega/\pi\sqrt{x^2 - 1/\omega g}$ for the range $|x| \geq \sqrt{1/\omega g}$. There are also interesting time dependent Euclidean solutions in the low density region $|x| \leq \sqrt{1/\omega g}$, which will be discussed later.

Action (1) has two notable deficiencies. First, the kinetic term is not in canonical form, signifying that the correct canonical field of the theory is not φ, and second, the coordinate x appears explicitly in the potential and therefore Poincare invariance seems to be broken explicitly. We remove both deficiencies. The first, following ref. [5], by expanding around the

classical solution, $\partial_x \varphi = \phi_0 + \partial_x \zeta / \sqrt{\pi}$ and changing coordinates $x \to \tau = \int^x dy/(\pi\phi_0)$. The resulting action for ζ is given by

$$S_E[\zeta] = \int dtd\tau \left\{ \frac{1}{2}(\dot\zeta^2 + \zeta'^2) - \frac{1}{2}\frac{g(\tau)\dot\zeta^2\zeta'}{1 + g(\tau)\zeta'} + \frac{1}{6}g(\tau)\zeta'^3 - \frac{1}{3}\frac{1}{g(\tau)^2} \right\}, \tag{2}$$

where $g(\tau)$ is a space dependent coupling parameter, which we define below, and the τ integration is over the limits $-\infty < \tau \le \tau_0 - \frac{\sigma}{2}$ and $\tau_0 + \frac{\sigma}{2} \le \tau < \infty$. In τ space, the low density region is given by $\tau_0 - \frac{\sigma}{2} < \tau < \tau_0 + \frac{\sigma}{2}$, so that τ_0 is the center of the low density region and σ is the width. The coupling parameter, defined over $-\infty < \tau \le \tau_0 - \frac{\sigma}{2}$ and $\tau_0 + \frac{\sigma}{2} \le \tau < \infty$, is given by $g(\tau) = (\pi^{3/2}\varphi_0(x))^{-1}$, and is found to be

$$\omega g(\tau) = 4\sqrt{\pi}g \frac{e^{-2\omega(\tau-\tau_0-\frac{\sigma}{2})}}{(1 - e^{-2\omega(\tau-\tau_0-\frac{\sigma}{2})})^2}, \tag{3}$$

for the range $\tau_0 + \frac{\sigma}{2} \le \tau < \infty$, with the obvious symmetric form in the range $-\infty < \tau \le \tau_0 - \frac{\sigma}{2}$. Notice that the coupling parameter blows up as $\tau \to \tau_0 \pm \frac{\sigma}{2}$; that is, at the boundaries of the low density region.

We turn now to correct the second deficiency of action (1) and restore Poincare invariance. We interpret, following ref.[7], the space dependent coupling parameter $g(\tau)$ as a field dependent coupling parameter $g(D)$. We further assume that the field D has a space dependent expectation value $\langle D \rangle$, such that $g(\tau) = g(\langle D \rangle)$. Furthermore, we impute the apparent lack of Poincare invariance of the action (2) solely to the space dependence of the expectation value $\langle D \rangle$. It turns out that, although not unique, a Poincare invariant action $S_E[\zeta, D]$ does exist and is not arbitrary. In its simplest form it is given by

$$S_E[\zeta, D] = \int d^2X \left\{ \frac{1}{2}(\nabla\zeta)^2 + \frac{1}{4}\frac{(\nabla\zeta)^2\nabla\zeta\cdot\nabla D}{1 - 1/(2\omega^2)g(D)\nabla\zeta\cdot\nabla D} - \frac{1}{48\omega^4}g(D)(\nabla\zeta\cdot\nabla D)^3 \right.$$
$$\left. + \frac{1}{32\omega^4}g(D)\frac{(\nabla\zeta\cdot\nabla D)^3}{1 - 1/(2\omega^2)g(D)\nabla\zeta\cdot\nabla D} - \frac{1}{24}\frac{1}{g^2(D)}\left[(\nabla D)^2 + 4\omega^2\right] \right\} \tag{4}$$

where the coupling $g(D)$ is given by

$$g(D) = 4\sqrt{\pi}g\frac{e^D}{(1 - e^D)^2}. \tag{5}$$

Thus g cannot be absorbed into a redefinition of D and scaled away completely from the effective action, although asymptotically, for $D \ll -1$ it is possible to scale it away.

Action (4) may look complicated, but it actually encodes simple and interesting dynamics. For $D \ll -1$ the coupling parameter $g(D)$ is negligibly small and our system reduces to two decoupled fields, one massless field ζ, and one superheavy field D. Note also that ζ has only derivative interactions and therefore no potential. As the value of D increases the interaction strength increases exponentially until the coupling blows up for $D = 0$, signaling the spontaneous generation of a boundary. The width of the strongly coupled region (the "wall") is $1/\omega$. The conclusion is therefore that the action (4) describes, effectively, a free massless scalar field ζ, moving in a bounded region of space-time.

The equations of motion derived from (4) are quite complicated but a complete and simple set of classical solutions may be found in a straightforward manner,

$$D_0 = AX_1 + BX_2 + C, \qquad\qquad \zeta_0 = c \tag{6}$$

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where A, B, C, c are real parameters and $A^2 + B^2 = 4\omega^2$. Each D_0 solution in (6) defines a line along which the coupling parameter blows up and a boundary is formed. The solutions we are interested in are a combination of two solutions for which the two boundaries are parallel to each other and at a constant fixed width. It is convenient to define the directions parallel and perpendicular to D_0, $\widehat{X}_\parallel = \frac{1}{2\omega}(AX_1 + BX_2)$ and $\widehat{X}_\perp = \frac{1}{2\omega}(-BX_1 + AX_2)$. We observe that Poincare invariance is not completely broken by the solutions (6). One translation in the $X_\perp$ direction remains unbroken. The energy E_0 of the classical solutions, which can be computed using standard methods, is found to vanish. In fact, the whole energy-momentum tensor $T_{\mu\nu}$ vanishes. Note that to compute correctly $T^{\mu\nu}$ one has to correctly couple gravity to the system, as done for example in [7], compute $T^{\mu\nu}$ in a curved background and then set space-time to be flat. Reduction of action (4) to collective field theory action is achieved by setting D to one of its possible expectation values $\langle D \rangle = D_0$ and identifying $X_\parallel$ with τ and $X_\perp$ with t. The effective action (4) reduces exactly to the corresponding action (2) for each classical solution.

Have we successfully restored Poincare invariance to our theory? It certainly seems so, but we have to be careful! Because the action (4) is Poincare invariant and the D_0 solutions break some of that invariance there should be zero-modes corresponding to the broken generators of Poincare (Euclidean) invariance. In our case they are a rotation, and a translation in the $X_\parallel$ direction. Indeed one finds that the expected zero-modes exist. For example, the wave function corresponding to the broken generator $\partial_{X_\parallel}$ is simply proportional to $\partial_{X_\parallel} D_0 = -2\omega$. The standard argument about symmetry restoration then says that each classical solution does break some of the symmetry, but the symmetry is restored by the summation over zero-modes. However, the standard argument holds only if all the zero-modes are normalizable. Of course, the correct measure, determined by the kinetic terms, has to be used to compute the normalization factor. When we compute the normalization factor N, for the $X_\parallel$ translation zero-mode, for example, we find that it is divergent $N^2 \sim \int dX_\parallel 1/g^2(X_\parallel)$. When a non-normalizable zero-mode appears the theory breaks up into separate superselection sectors parametrized by the value the broken generator takes in each sector. In particular this would mean that Poincare invariance was not really fully restored in our theory (see, for example, a discussion of For example, the wave function corresponding to the broken Note that the divergence of N comes from the weak coupling region, and therefore to achieve actual Poincare invariance restoration we need to regulate the weak coupling behavior of D in some way. At the moment we cannot offer a conclusive opinion on whether and how the weak coupling behavior of D can be modified and regulated. We can look for clues by understanding how a higher-dimensional string theory handles a similar challenge. Let us look, for example, at the 5-dimensional extremal black hole solution of the heterotic string [9]

$$ds^2 = -Qdt^2 + (1 + \frac{Q}{r^2})(dr^2 + r^2 d\Omega_3^2), \qquad e^{2(D-D_0)} = 1 + \frac{Q}{r^2} \qquad (7)$$

where $H = Q\epsilon_3$ is the solution for field strength of the antisymetric tensor B and D is the dilaton. In the "throat" region, $\frac{Q}{r^2} >> 1$ the solution is approximately

$$ds^2 \sim -Qdt^2 + Qd\tau^2 + Qd\Omega_3^2 \qquad D - D_0 \sim -\tau \qquad (8)$$

where $\tau \sim \ln r$. Many other similar examples in different dimensions can be found in the review [10]. It is important to note that the dilaton (the analog of our field D) in the exact solution is asymptotically constant. It varies linearly only in the "throat" region. The translation zero-modes corresponding to the broken translation generators in the background of the classical solution (7) are normalizable since the would-be weak coupling divergence is regulated by the constant non-zero asymptotic value of the dilaton. Consider the expansion of the heterotic

string effective action around the classical solution in the "throat" region [11]. Because the geometry of this region is that of $M_2 \times S_3$, the light fields can be described by an effective two dimensional field theory in (t, τ) space. This theory is of course not Poincare invariant because the dilaton has a space dependent expectation value. The coupling parameter of the theory varies in space for the same reason. The light fields of the 2-d theory are just the modes of the antisymmetric tensor which in this case can be described by one derivatively coupled scalar field, the axion. The similarity between our 2-d theory and the one associated with regions of linear dilaton solutions of the heterotic string suggests a physical way to regulate the weak coupling divergence by modifying the behavior of D from linear to constant asymptotically.

3   Stringy Non-perturbative effects

As mentioned previously, in addition to the static, high density, solution of the collective field theory equations of motion discussed in the previous section, there are interesting time-dependent Euclidean solutions in the low density region. In the effective field theory the low density region is formally a region of infinite coupling and the important degrees of freedom are not smooth excitations of the fields, but rather single matrix eigenvalues, corresponding to singular field configurations (see also [12]). To expose the important physics in the low density region we separate one discrete eigenvalue from the continuum and look at it's effective dynamics,

$$L_E[\lambda_0; \varphi] = \frac{1}{2}\dot{\lambda}_0^2 + \frac{1}{2}\omega^2(\frac{1}{\omega g} - \lambda_0^2) + \int dx \frac{\varphi'}{(x - \lambda_0)^2} + \int dx \left\{ \frac{\dot{\varphi}^2}{2\varphi'} + \frac{\pi^2}{6}\varphi'^3 + \frac{1}{2}\omega^2(\frac{1}{\omega g} - x^2)\varphi' \right\}. \quad (9)$$

The third term in this expression represents the mutual interaction of the discrete eigenvalue with the continuum eigenvalues, which are collectively described using the classical solution ϕ_0. We obtain the Euclidean equations of motion for λ_0 by variation of (9), they are given in the small g limit simply by

$$\ddot{\lambda}_0 + \omega^2 \lambda_0 = 0; \;\; -1/\sqrt{\omega g} < \lambda_0 < 1/\sqrt{\omega g} \qquad \ddot{\lambda}_0 = 0; \;\; \lambda_0 = \pm 1/\sqrt{\omega g}. \quad (10)$$

We also impose the following boundary conditions, $\lambda_0(t \to -\infty) = \pm 1/\sqrt{\omega g}$ and, independently, $\lambda_0(t \to +\infty) = \pm 1/\sqrt{\omega g}$. There are two static solutions to (10) which satisfy this boundary condition,

$$\lambda_0^{(\pm)} = \pm 1/\sqrt{\omega g} \sin \omega(t - t_1) \;\; ; \;\; t_1 - \frac{\pi}{2\omega} \leq t \leq t_1 + \frac{\pi}{2\omega}, \quad (11)$$

representing tunneling of single eigenvalues across the potential barrier in the low density region.

The partition function associated with the theory discussed above can be written as a sum over different instanton sectors and after some lengthy analysis [13], using a dilute gas approximation, we arrive at the following general result

$$
\begin{aligned}
Z &= \int [d\varphi] e^{-S_\varphi[\varphi]} \sum_{q=0}^{\infty} \frac{1}{q!} \mathcal{M}^q \prod_{i=1}^{q} \int dt_i \sum_{\{k_i\}} \prod_{j=1}^{q} e^{-S_I^{(k_j)}[\varphi; t_j]} \\
&= \int [d\varphi] e^{-S_\varphi[\varphi]} \sum_{q=0}^{\infty} \frac{1}{q!} \left\{ \mathcal{M} \int dt_1 \left(e^{-S_I^{(+)}[\varphi; t_1]} + e^{-S_I^{(-)}[\varphi; t_1]} \right) \right\}^q.
\end{aligned} \quad (12)
$$

The sum over q is now an exponential, so that $Z = \int [d\varphi] e^{-S_{eff}[\varphi]}$, where $S_{eff}[\varphi] = S_\varphi[\varphi] + \Delta S[\varphi]$ is the effective action with the instanton effects systematically incorporated, and

$$\Delta S[\varphi] = \mathcal{M} \int dt_1 \left\{ e^{-S_I^{(+)}[\varphi; t_1]} + e^{-S_I^{(-)}[\varphi; t_1]} \right\} \quad (13)$$

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is the associated change in the action. The action $S_I^{(\pm)}$ is given by

$$S_I^{(\pm)}[\varphi; t_j] = \int_{t_j - \frac{\pi}{2\omega}}^{t_j + \frac{\pi}{2\omega}} dt \int dx \left\{ \frac{\varphi'(x,t)}{(x - \lambda_0^{(\pm)}(t - t_j))^2} - \frac{\varphi'(x,t)}{(x - \lambda_\emptyset^{(\pm)}(t - t_j))^2} \right\}. \tag{14}$$

where

$$\lambda_\emptyset^{(\pm)}(t; t_1) = \left\{ \begin{array}{ll} \mp 1/\sqrt{\omega g} & ; \quad t_1 - \frac{\pi}{2\omega} \leq t < t_1 \\ \pm 1/\sqrt{\omega g} & ; \quad t_1 < t \leq t_1 + \frac{\pi}{2\omega} \end{array} \right. . \tag{15}$$

The quantity $\mathcal{M}$ is a dimensionful parameter that sets the basic strength for induced non-perturbative interactions

$$\mathcal{M} = \omega \sqrt{\frac{\pi}{2g}} e^{-\frac{\pi}{2g}}. \tag{16}$$

A full analysis to find the induced operators in collective field theory action was carried out [13]. We give only the final result for the simplest induced operator.

$$\Delta S[\zeta] = 2\omega g^{-1/6} e^{-\frac{\pi}{2g}} \int dt e^{-\frac{2\sqrt{2}}{3\omega} \zeta'(\tau_0, t)}. \tag{17}$$

Note that the the induced operator contains only a t integration which appears because of the existence of a normalizable t translation zero-mode. Similarly, once a good regularization procedure is found to render the two other zero-modes normalizable, the induced operators due to our instantons would be integrated against a Poincare-invariant integration measure and therefore themselves be Poincare invariant completions of the computed operators we discussed above.

4 Concluding remarks and Outlook

We described a method for the evaluation of stringy non-perturbative effects and their systematic inclusion in the form of induced operators into an effective action. We have not yet completed the calculation due to a missing regularization procedure which has to be developed. When we successfully find a regularization scheme we may turn to the evaluation of similar effects in the supersymmetric effective theory presented in [14] and also in higher dimensional theories, if these theories contain strong coupling regions that are effectively two dimensional, as in the example presented in section 2. It would be interesting to find out in which form the operators in the supersymmetric theory appear. One possibility is that stringy non-perturbative effects induce supersymmetric operators, which may affect the pattern of supersymmetry breaking, the other possibility is that the induced operators break supersymmetry. Preliminary indications suggest that the latter possibility, but final conclusions have to We described a method for

5 Acknowledgment

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